



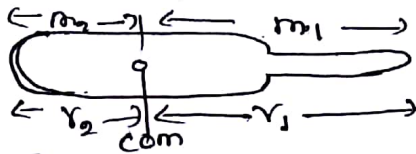
CENTRE OF MASS (C.O.M) :-

Point $\uparrow$ about which mass distributed uniformly $\downarrow$

$$m\vec{r} \Rightarrow \text{constant} \leftarrow \text{moment of mass}$$

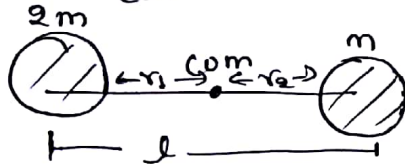
$$m_1 r_1 = m_2 r_2 = \text{constant}$$

eg:-



$$\Rightarrow m_1 r_1 = m_2 r_2$$

eg:-



$$r_1 + r_2 = l \quad \text{--- (1)}$$

$$m_1 r_1 = m_2 r_2$$

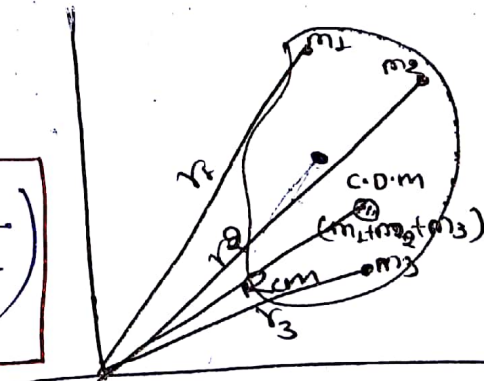
$$\frac{r_1}{r_2} = \frac{m_2}{m_1}$$

$$r_1 = \left(\frac{m_2}{m_1 + m_2} \right) l$$

$$m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \dots$$

$$\Rightarrow (m_1 + m_2 + m_3 + \dots) \vec{R}_{cm}$$

$$\vec{r}_{cm} = \left(\frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \dots}{m_1 + m_2 + m_3 + \dots} \right)$$



for two body

$$\vec{R}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

$$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

$$\vec{p}_{cm} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2}$$

$$\vec{v}_{cm} = \vec{u}_{cm} + \vec{a}_{cm} t$$

$$\vec{s}_{cm} = \vec{u}_{cm} t + \frac{1}{2} \vec{a}_{cm} t^2$$

$$v_{cm}^2 = u_{cm}^2 + 2 \vec{a}_{cm} \vec{s}$$

#

$$\vec{r}_1 = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}$$

$$\vec{r}_2 = x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k}$$

$$\vec{R}_{cm} = x_{cm} \hat{i} + y_{cm} \hat{j} + z_{cm} \hat{k}$$

$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{\sum m x}{\sum m} = \frac{\int x \cdot dm}{\int dm}$
$y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{\sum m y}{\sum m} = \frac{\int y \cdot dm}{\int dm}$
$z_{cm} = \frac{m_1 z_1 + m_2 z_2}{m_1 + m_2} = \frac{\sum m z}{\sum m} = \frac{\int z \cdot dm}{\int dm}$

From geometry :-

1-D : $x_{cm} = \frac{l_1 x_1 + l_2 x_2}{l_1 + l_2}$, $y_{cm} = \frac{l_1 y_1 + l_2 y_2}{l_1 + l_2}$

$$z_{cm} = \frac{l_1 z_1 + l_2 z_2}{l_1 + l_2}$$

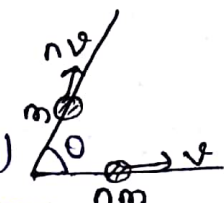
2-D : $x_{cm} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$, $y_{cm} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

$$z_{cm} = \frac{A_1 z_1 + A_2 z_2}{A_1 + A_2}$$

3-D : $x_{cm} = \frac{V_1 x_1 + V_2 x_2}{V_1 + V_2}$, $y_{cm} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2}$

$$z_{cm} = \frac{V_1 z_1 + V_2 z_2}{V_1 + V_2}$$

Q₁: Find V_{cm} ?

$$\vec{V}_{cm} = \frac{nmv\hat{i} + m \cdot nv(\cos\theta\hat{i} + \sin\theta\hat{j})}{nm+m}$$


$$\vec{V}_{cm} = \frac{nv(1 + \cos\theta\hat{i} + \sin\theta\hat{j})}{n+1}$$

$$= \frac{nv}{n+1} [(1+\cos\theta)\hat{i} + \sin\theta\hat{j}]$$

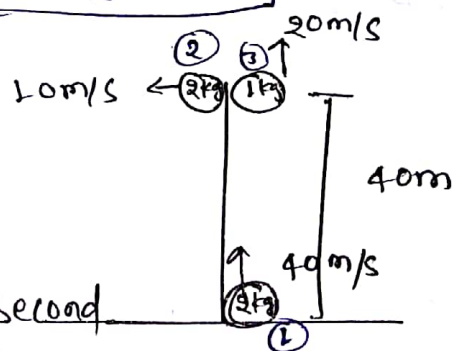
$$V_{cm} = \frac{nv}{n+1} \sqrt{1 + \cos^2\theta + 2\cos\theta + \sin^2\theta}$$

$$= \frac{nv}{n+1} \sqrt{2(1+\cos\theta)} \Rightarrow \boxed{\frac{2nv\cos\frac{\theta}{2}}{n+1}} \text{ Ans.}$$

Q₂: Find displacement of centre of mass after one second.

(i) V_{cm} after 1 second

(ii) position of C.O.M. after 2 second



$$x_{cm} = 0$$

$$z_{cm} = 0$$

$$y_{cm} = \frac{2 \times 0 + 2 \times 40 + 1 \times 40}{2+2+1} = \boxed{24m}$$

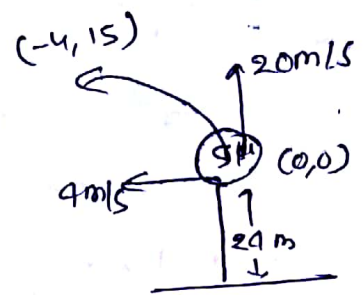
$$\vec{V}_{cm} = \frac{2 \times 40\hat{j} + 2(-10\hat{i}) + 20\hat{j} \times 1}{5}$$

$$= \frac{100\hat{j} - 20\hat{i}}{5} = \boxed{(20\hat{j} - 4\hat{i}) \text{ m/s}}$$

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3}{m_1+m_2+m_3}$$

$$= \frac{2 \times g + 2 \times g + 1 \times g}{5} = g \downarrow$$

$$\begin{aligned}
 x &= u \cos \alpha \cdot t \\
 &= 4 \times 1 = 4 \text{ m} \\
 y &= 20 \times 1 - \frac{1}{2} \times 10 \times 1^2 \\
 &= 20 - 5 = 15 \text{ m}
 \end{aligned}$$



(i) Displacement

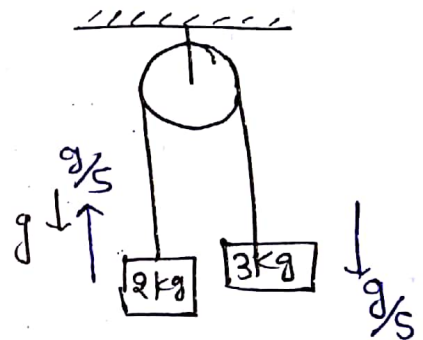
$$\sqrt{(4)^2 + (15)^2} = \sqrt{241}$$

(ii) $\vec{v} = -4\hat{j} + 10\hat{j}$
 $= \sqrt{(-4)^2 + (10)^2} = \sqrt{116}$

(iii) Position from earth
after 2 second
 $(-8 \text{ m}, 44 \text{ m})$.

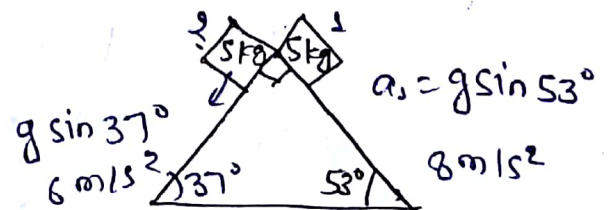
Q = $a = \frac{3g - 2g}{5} = \frac{g}{5}$

$$\begin{aligned}
 (a_{cm})_{\downarrow} &= \frac{2 \times (-\frac{g}{5}) + 3 \times \frac{g}{5}}{5} \\
 &= \frac{g}{25} \Rightarrow \frac{2}{5} \text{ m/s}^2
 \end{aligned}$$



Q = find $|\vec{a}_{cm}|$

$$\begin{aligned}
 \vec{a}_{cm} &= \frac{5\vec{a}_1 + 5\vec{a}_2}{5+5} \\
 \vec{a}_{cm} &= \frac{\vec{a}_1 + \vec{a}_2}{2}
 \end{aligned}$$



$$\frac{1}{2} |\vec{a}_1 + \vec{a}_2| \Rightarrow \frac{1}{2} \sqrt{a_1^2 + a_2^2} = \frac{1}{2} \sqrt{(6)^2 + (8)^2}$$

$$\frac{1}{2} \times 10 = 5 \text{ m/s}^2$$

Q. 3- $|\vec{a}_2|$

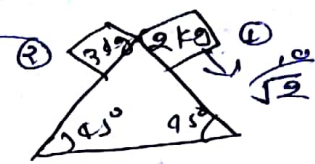
$|\vec{a}_{cm}| = \frac{2\vec{a}_1 + 3\vec{a}_2}{5}$

$\frac{1}{5} |2\vec{a}_1 + 3\vec{a}_2| \Rightarrow \frac{1}{5} \sqrt{9a_2^2 + 4a_1^2}$

$\Rightarrow \frac{1}{5} \sqrt{9 \times \frac{g^2}{2} + 4 \times \frac{g^2}{2}}$

$\Rightarrow \frac{1}{5} \sqrt{\frac{13g^2}{2}}$

$\Rightarrow \frac{1}{5} \sqrt{650} \Rightarrow \frac{1}{5} \sqrt{25 \times 26} = \sqrt{26} \text{ m/s}^2$



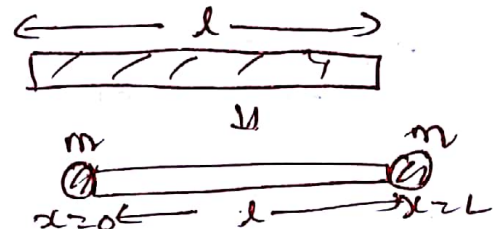
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$X_{cm} = \frac{m \times 0 + m \times l}{m + m}$

$X_{cm} = \frac{ml}{2m}$

$X_{cm} = l/2$

Homogeneous



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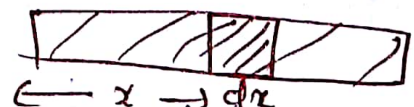
Homogeneous

Linear mass density

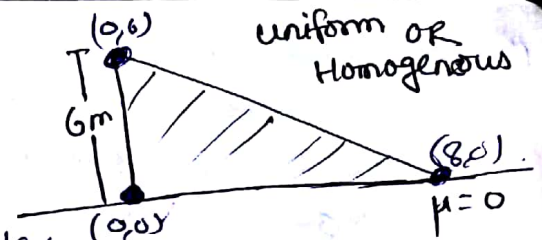
$\lambda = \frac{m}{l}$ ①

$dm = \lambda \cdot dx$

$X_{cm} = \frac{\int x dm}{\int dm} \Rightarrow \frac{\int_0^l x \cdot \lambda dx}{\int_0^l \lambda dx} = \frac{l}{2}$



Q₁₁ find velocity at which released body strike on earth?



$$x_{cm} = \frac{m \times 0 + m \times 8 + m \times 0}{3m} = \frac{8}{3} \text{ meter}$$

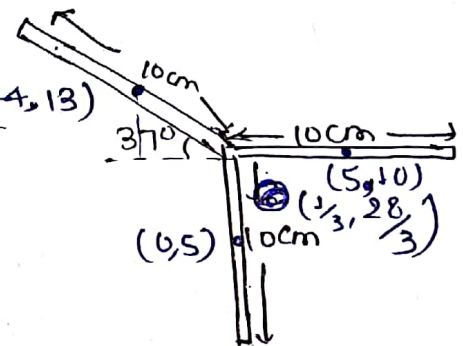
$$y_{cm} = \frac{m \times 0 + m \times 0 + m \times 6}{3m} = 2 \text{ meter}$$

$$v = \sqrt{2gy_{cm}}$$

$$v = \sqrt{2 \times 9 \times 2}$$

$$v = 2\sqrt{10} \text{ m/s}$$

Q₁₂ speed at which released body strike on earth.



$$x_{cm} = \frac{l_1 x_1 + l_2 x_2 + l_3 x_3}{l_1 + l_2 + l_3}$$

$$= \frac{10 \times 0 + 10 \times 5 + 10 \times (-4)}{10 + 10 + 10}$$

$$= \frac{1}{3} \text{ meter}$$

$$y_{cm} = \frac{l_1 y_1 + l_2 y_2 + l_3 y_3}{l_1 + l_2 + l_3} \Rightarrow \frac{10 \times 5 + 10 \times 10 + 13 \times 10}{30}$$

$$= \frac{28}{3}$$

$$v = \sqrt{2 \times 9 y_{cm}} = \sqrt{2 \times 10 \times \frac{28}{3}} = 4\sqrt{\frac{35}{3}}$$

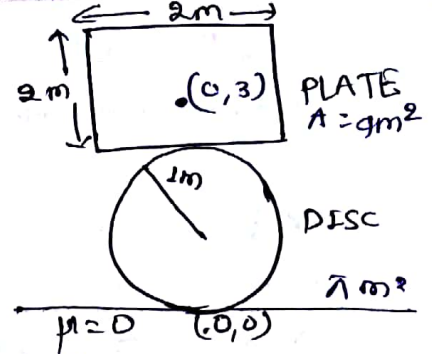
Q₁₁:- find C.O.M. & velocity & which body strike on earth?

$$x_{cm} = 0$$

$$y_{cm} = \frac{\pi \times 1 + 4 \times 3}{\pi + 4}$$

$$y_{cm} = \frac{(\pi + 12)}{(\pi + 4)}$$

position of C.O.M. = $\left(0, \frac{\pi + 12}{\pi + 4} \right)$



$$v_{cm} = \sqrt{2g \left(\frac{\pi + 12}{\pi + 4} \right)}$$

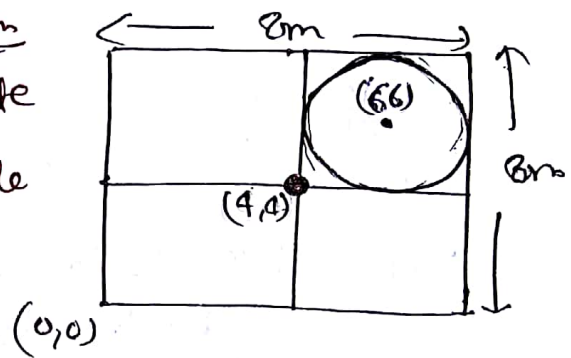
Q₁₂:- find position of C.O.M.

$$A_1 = 64 \text{ m}^2 \text{ Area of plate}$$

$$A_2 = 4\pi \text{ m}^2 \text{ Area of circle}$$

$$x_{cm} = \frac{64 \times 4 - 4\pi \times 6}{64 - 4\pi}$$

$$y_{cm} = \frac{64 \times 4 - 4\pi \times 6}{64 - 4\pi}$$

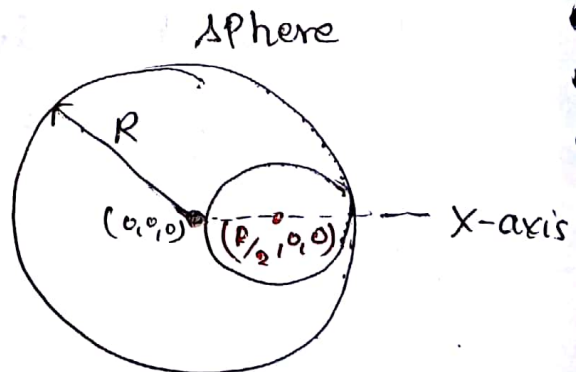


Q₁₃:- position of C.O.M.?

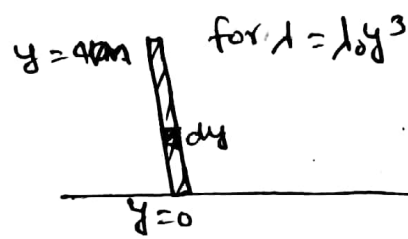
$$x_{cm} = \frac{v_1 x_1 - v_2 x_2}{v_1 - v_2}$$

$$= \frac{0 - \frac{v_1}{8} \frac{R}{2}}{v_1 - \frac{v_1}{8}}$$

$$= \frac{R}{14}$$



Q₁₀ - find speed at which released body strike on earth?

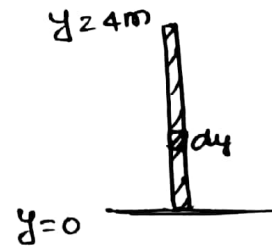


$$dm = \lambda \cdot dy$$

$$= \lambda_0 y^3 dy$$

$$y_{cm} = \frac{\int y \cdot dm}{\int dm} \Rightarrow \frac{\int_0^4 y \cdot \lambda_0 y^3 dy}{\int_0^4 \lambda_0 y^3 dy} = \frac{16}{5} m$$

Q₁₁ - Linear mass density varies linearly from $y=0$ to $y=4m$ as $\lambda = 2 + 3y$. find position (kgm⁻¹)



of C.M. & speed of released body when it strike on earth

$$dm = \lambda \cdot dy \Rightarrow (2 + 3y) dy$$

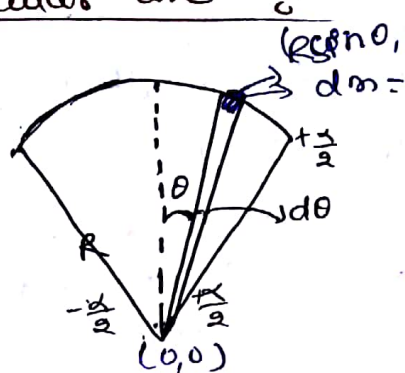
$$y_{cm} = \frac{\int_0^4 y \cdot dm}{\int_0^4 dm} = \frac{\int_0^4 y \cdot (2 + 3y) dy}{\int_0^4 (2 + 3y) dy}$$

Position of COM on circular arc :-

$$dm = \lambda \cdot d\theta$$

$$= \frac{m}{\alpha} \cdot d\theta$$

$$X_{cm} = \frac{\int_{-\alpha/2}^{+\alpha/2} R \sin \theta \cdot \frac{m}{\alpha} \cdot d\theta}{\int_{-\alpha/2}^{+\alpha/2} \frac{m}{\alpha} d\theta} = 0$$



$$Y_{cm} = \frac{\int_{-\alpha/2}^{+\alpha/2} R \cos \theta \cdot \frac{m}{\alpha} \cdot d\theta}{\int_{-\alpha/2}^{+\alpha/2} \frac{m}{\alpha} d\theta} \Rightarrow \frac{R \sin \frac{\alpha}{2}}{\alpha}$$

$$Y_{cm} = \frac{R \sin \frac{\alpha}{2}}{\frac{\alpha}{2}} \quad \text{Ans.}$$

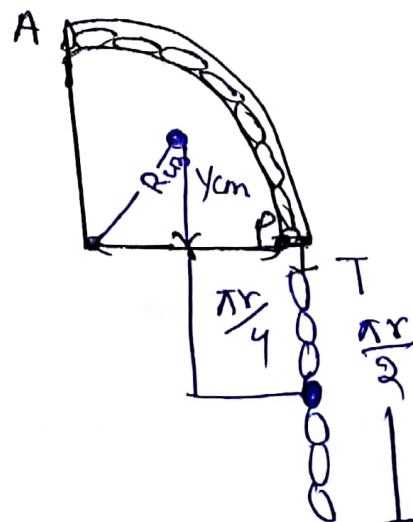
Semicircle $\Rightarrow$ $R_{cm} = \frac{R \sin \frac{\alpha}{2}}{\frac{\alpha}{2}}$

$$R_{cm} = \frac{R \sin 90}{\frac{\pi}{2}} = \frac{2R}{\pi}$$



∴ speed of chain when its end A passes through point P.

$$R_{cm} = \frac{r \sin 45^\circ}{\frac{\pi}{4}} = \frac{4r}{\pi \sqrt{2}} = \frac{2\sqrt{2}r}{\pi}$$



$$Y_{cm} = R_{cm} \sin 45^\circ$$

$$= \frac{2\sqrt{2}r}{\pi} \times \frac{1}{\sqrt{2}} \Rightarrow \frac{2r}{\pi}$$

$$\frac{1}{2}mv^2 = mg\left(\frac{2r}{\pi} + \frac{\pi r}{4}\right)$$

$$v = \sqrt{2gr\left(\frac{2}{\pi} + \frac{\pi}{4}\right)}$$

Q:- Speed of chain when its end A passes through Point P.

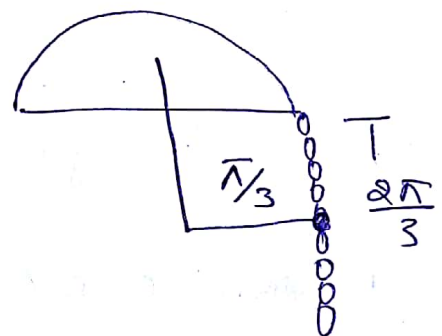
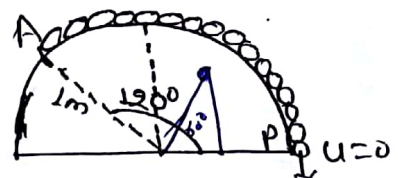
$$r_{cm} = \frac{1 \cdot \sin 60^\circ}{\frac{\pi}{3}} = \frac{3\sqrt{3}}{2\pi}$$

$$Y_{cm} = R_{cm} \sin 60^\circ$$

$$= \frac{3\sqrt{3}}{2\pi} \cdot \frac{\sqrt{3}}{2}$$

$$Y_{cm} = \frac{9}{4\pi}$$

$$v = \sqrt{2g\left(\frac{9}{4\pi} + \frac{\pi}{3}\right)}$$

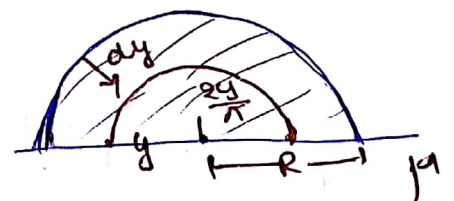


Semi-circular plate:-

$$① \delta = \frac{m}{\pi R^2}$$

$$dm = \delta \cdot \pi y \cdot dy$$

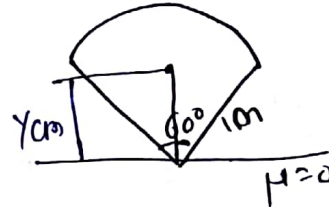
$$② Y_{cm} = \frac{\int_0^R \frac{\pi y}{\pi} \cdot \delta \cdot \pi y \cdot dy}{\int_0^R \delta \cdot \pi y \cdot dy} = \frac{\frac{\pi}{\pi} \left[\frac{y^3}{3} \right]}{\frac{y^2}{2}} = \frac{2}{3} \left(\frac{R}{2} \right)$$



$$Y_{cm} = \frac{\frac{\text{arc}}{\frac{2R}{\pi}}}{\frac{2}{3} \left(\frac{2R}{\pi} \right)}$$

$$\boxed{Y_{cm} = \frac{2}{3} (\text{arc})}$$

Speed at which released body strike on earth.



$$Y_{cm} = R_{cm} = \frac{2}{3} \left[\frac{L \cdot \sin 30^\circ}{\frac{\pi}{6}} \right]$$

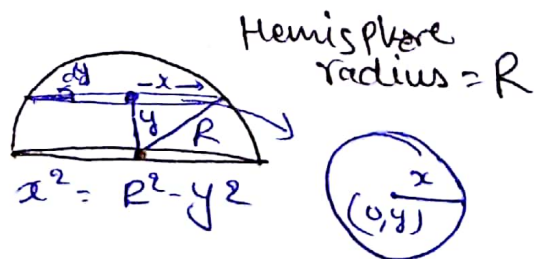
$$Y_{cm} = \frac{2}{3} \times \frac{1}{\frac{\pi}{6}} = \frac{2}{\pi}$$

$$v = \sqrt{2g \left(\frac{2}{\pi} + \frac{\pi}{6} \right)}$$

$$v = \sqrt{2g \left(\frac{12 + \pi^2}{6\pi} \right)}$$

Position of C.O.M.?

$$I = \frac{2}{3} \pi R^3$$



$$X_{cm} = 0$$

$$Y_{cm} = \frac{\int y \cdot \rho \cdot \pi x^2 dy}{\int \rho \cdot \pi x^2 dy} \Rightarrow \frac{\int_0^R y(R^2 - y^2) dy}{\int_0^R (R^2 - y^2) dy}$$

$$\boxed{Y_{cm} = \frac{3R}{8}}$$

Ans.

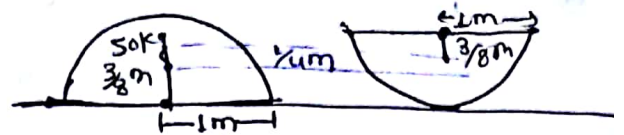
Q₁₁ Required work in toppling?

C.O.M. of 1st = $\frac{3}{8}$

$$\downarrow - \left(\frac{3}{8} + \frac{3}{8} \right) = \frac{1}{4}$$

$$W = 50 \cdot 10 \times \frac{1}{4}$$

$$W = 125 \text{ J}$$



Q₁₂:- C.O.M. of HEMISPHERICAL SHELL?

$$\delta = \frac{M}{2\pi R^2}$$

$$dm = \delta \cdot 2\pi x \cdot R d\theta$$

$$dm = \delta \cdot 2\pi R^2 \cdot \sin\theta d\theta$$

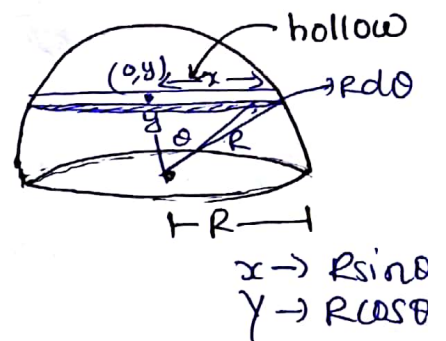
$$x_{cm} = 0$$

$$y_{cm} = \frac{\int_0^{\pi/2} R \cos\theta \cdot \delta \cdot 2\pi R^2 \sin\theta d\theta}{\int_0^{\pi/2} \delta \cdot 2\pi R^2 \sin\theta d\theta}$$

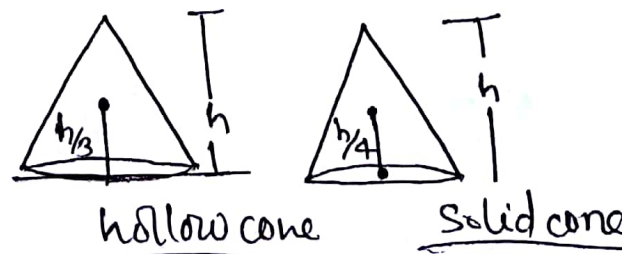
$$= R \int_0^{\pi/2} \frac{\cos\theta}{\sin\theta} d\theta \Rightarrow R \left(\frac{1}{2} \right)$$

$$\Rightarrow \frac{R}{2} [\sin\theta]_0^{\pi/2}$$

$$\Rightarrow \boxed{\frac{R}{2}}$$



centre of mass always toward more mass



Q₄: Position of C.O.M after cuts cone from hemisphere?

$$V_2 = \frac{1}{3} \pi \frac{R^2}{4} \times \frac{R}{2}$$

$$V_1 = \frac{2}{3} \pi R^3$$

$$V_2 = \frac{1}{3} \cdot \pi \cdot \frac{R^2}{4} \cdot \frac{R}{2}$$

$$= \frac{2}{3} \cdot \frac{\pi R^3}{16}$$

$$= \frac{V_1}{8}$$

$$x_{cm} = 0$$

$$y_{cm} = \frac{V_1 y_1 - V_2 y_2}{V_1 - V_2} = \frac{V_1 \times \frac{3R}{8} - \frac{V_1}{8} \times \frac{R}{4}}{V_1 - \frac{V_1}{8}} = \boxed{\frac{11R}{28}}$$

Q₅: Speed at which released body strike on earth for sphere

$$V_2 = \frac{4}{3} \pi R^3$$

$$x_2 = 0, y_2 = 5R$$

for cone

$$V_1 = \frac{1}{3} \pi R^2 \cdot 4R$$

$$V_1 = V_2 = V$$

$$x_1 = 0$$

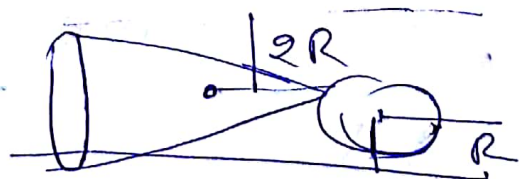
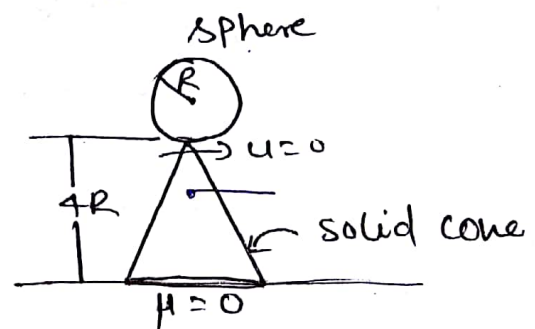
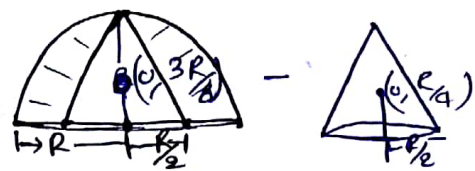
$$y_1 = R$$

$$x_{cm} = 0$$

$$y_{cm} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2} \Rightarrow \frac{V \times 5R + V \cdot R}{V + V} = 3R$$

$$V = \sqrt{4 \times gR}$$

$$\boxed{V = \sqrt{4gR} \text{ Ans}}$$



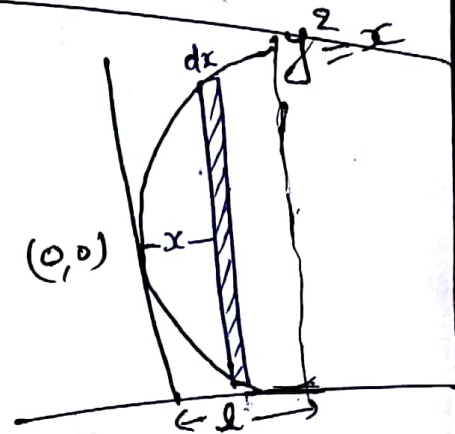
Q₁₁: position of C.O.M.

$\rho = \text{constant (mass per unit Volume)}$

$$dm = \rho \cdot y dx$$

$$x_{cm} = \frac{\int x \cdot \rho \cdot y dx}{\int \rho \cdot y \cdot dx}$$

$$\frac{\int_0^l x^{3/2} dx}{\int_0^l x^{1/2} dx} \Rightarrow \frac{\frac{2}{5} l^{5/2}}{\frac{2}{3} l^{3/2}} \Rightarrow \frac{3 \times \frac{2}{5} l^{5/2}}{5 \times \frac{2}{3} l^{3/2}} = \boxed{\frac{3l}{5}} \text{ Ans.}$$



~~When~~ External force = 0 means $a_{cm} = 0$

$$a_{cm} = 0$$

$$m_1 a_1 + m_2 a_2 = 0$$

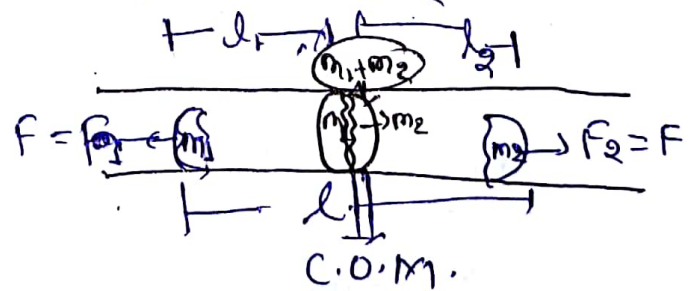
$$\vec{F}_1 + \vec{F}_2 = 0$$

$$\vec{F}_1 = -\vec{F}_2 \Rightarrow$$

$$\left(\vec{F}_1 \right) = \left(\vec{F}_2 \right)$$

$$m_1 a_1 = m_2 a_2$$

Q₁₂: - If fragments cover distance l relative to each other in time t then find internal force "F".



$$\text{Work done} = F l_1 + F l_2 = F l \quad \text{--- (1)}$$

$$\frac{l_1}{l_2} = \frac{\frac{1}{2} a_1 t^2}{\frac{1}{2} a_2 t^2} \Rightarrow \frac{l_1}{l_2} = \frac{F m_2}{F \cdot m_1}$$

$$\boxed{l_1 = \frac{m_2}{m_1 + m_2} \cdot l}, \quad \boxed{l_2 = \frac{m_1}{m_1 + m_2} \cdot l}$$

$$\Rightarrow \boxed{\frac{l_1}{l_2} = \frac{m_2}{m_1}} \Rightarrow \boxed{m_1 l_1 = m_2 l_2} \Rightarrow \text{const}$$

$$\# l_1 = \frac{1}{2} \frac{F}{m_1} t^2 \Rightarrow \frac{m_2 l}{m_1 + m_2} = \frac{F}{2m_1} l^2$$

$$F = \frac{2m_1 m_2 l}{(m_1 + m_2) t^2} \Leftrightarrow \left[\frac{2}{\left(\frac{1}{m_1} + \frac{1}{m_2} \right)} \right] \frac{l}{t^2}$$

for n part

$$F = \left(\frac{n}{\left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_n} \right)} \right) \frac{l}{t^2}$$

$F_{ext} = 0$
 $\downarrow$
 $a_{cm} = 0$

$\vec{v}_{cm} = \text{const}$ $\vec{v}_{cm} = 0$

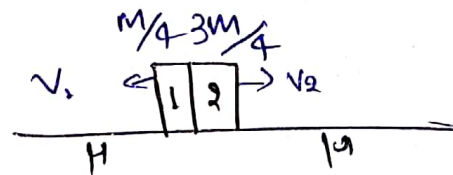
$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = 0$$

$$m_1 \vec{v}_1 = -m_2 \vec{v}_2$$

$$\left(m_1 \vec{v}_1 \right) = \left(m_2 \vec{v}_2 \right)$$

recoiling

Q. After Internal explosion in mass ratio 1:3. if 1st part cover distance 's' then second part displaced by
(a) s (b) 3s (c) 9s (d) none



$$\frac{S_1}{S_2} = \frac{v_1^2}{v_2^2} \Rightarrow \begin{aligned} 0 &= v_1^2 - 2HgS_1 \\ 0 &= v_2^2 - 2HgS_2 \end{aligned}$$

$$\frac{m/4 v_1}{3m/4 v_2} = -$$

$$\frac{v_1}{v_2} = 3$$

$$\frac{S_1}{S_2} = (3)^2$$

$$\boxed{S_2 = \frac{S_1}{9}} \Rightarrow \boxed{S_2 = \frac{s}{9}}$$

Q. find $a_1 = ?$



$$m_1 \vec{a}_1 + m_2 \vec{a}_2 + m_3 \vec{a}_3 = 0$$

$$1 \times \vec{a}_1 + 2 \times \hat{i} + 3 \times 2\hat{i} = 0$$

$$\vec{a}_1 = -8\hat{i} \quad \Rightarrow \quad a_1 = 8 \text{ m/s}^2 \quad \leftarrow$$